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Multi-affine analysis of typical currency exchange rates

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Abstract. For foreign currency exchange rates, multi-affine analysis can put quantitatively into evidence the differences between correlated (daily closing market) values and random walks in time dependent data. The $H(q)$ spectrum is presented and discussed here for the USD/DEM and JPY/USD exchange rates. The time-evolution of these ratios is found to be multi-affine. The $h(\gamma)$-curve describing the hierarchy of exponents is numerically obtained. Our findings suggest that the modelling of exchange rate time-evolution from day to day is possible within the framework of modern statistical physics and related to models of turbulence in the physics of fluids. Finally, we argue that there is a multiplicity of information levels in the foreign exchange market such that the "efficient market theory" is a crude oversimplification indeed.

PACS. 05.40.+j Fluctuation phenomena, random processes, and Brownian motion – 01.75.+m Science and society

1 Introduction

It has been recently debated whether physicists could be involved in financial affairs, whether physical ideas, methods and models would be beneficial to financial life [1]. The best answer to the first question is outside the scope of this journal, but the second question can find an interesting answer if ideas, methods and models are proved to be sound not only with respect to data analysis but also predictability – a key element in economic planning. Text books presenting high level or general mathematics for financial derivatives and the like exists already [2]. However, much should be expected from the basic physical concepts, *e.g.* scaling, fractal phenomena, self-organization, *etc.* In fact, the scope of statistical physics has widened tremendously and its technical methods have already penetrated into a number of fields beyond the traditional boundaries of physics, *i.e.* towards biology, ecology, geology, meteorology, *etc.* [3,4].

Problems in economics and finance have started to attract the interest of the statistical physics community. These problems concern *e.g.* the minimization of the risk for put and call options [5], the analysis of data near economic crashes like that of Oct. 19, 1987 [6–9], the search for forecasting [10], the analysis [11] and modeling [12] of company growth...

A fundamental problem is the existence or not of long-range correlations in currency exchange rates, *e.g.* the USD/DEM ratio, or in economic indices like the Standard and Poor 500 (S&P500) index.

Various statistical methods have been used in order to measure temporal correlations in financial data. Traditional methods (like spectral methods) have corroborated that there is evidence that the Brownian motion idea is only approximately right [13,14]. Through a data analysis based on the Lévy statistics, Mantegna and Stanley [15] have shown the existence of a power law distribution of returns in the *Standard and Poor* index (S&P500). Wavelet analysis of the same S&P500 index [16] and of the USD/DEM currency exchange rate [17] have also demonstrated the emergence of hidden substructures in such economic signals. These investigations suggest that the economic data evolve as *self-affine* functions on long time scales, *i.e.* from a few days to more than one year.

For any (discrete) time-dependent self-affine function $y(t)$, we can choose a particular point on the signal and rescale its neighborhood by a factor b using the roughness (or Hurst [18]) exponent H by considering the signal $b^{-H}y(bt)$. For the correct exponent value H, the signal obtained should be indistinguishable from the original one, *i.e.*

$$y(t) \sim b^{-H} y(bt). \tag{1}$$

An exponent $H < 1/2$ involves an *antipersistent* behavior while $H > 1/2$ means a *persistent* signal [18]. The simple Brownian motion is characterized by $H = 1/2$ and white noise by $H = 0$.

In the present report, we are interested in the "excursion" of the signal y after any time period τ, *i.e.* in the positive values $|y(t) - y(t+\tau)|$. Neglecting any bias or trend in the signal y, the "excursion" is simply related to the variance σ of the signal around its average value. For a self-affine signal, we have

$$\sigma \sim \tau^{H}. \tag{2}$$

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More precisely, temporal correlations exist for $H \neq 1/2$. Indeed, the correlation C of future increment $y(t) - y(0)$ with past increment $y(0) - y(-t)$ is given by

$$C = \frac{\langle (y(0) - y(-t))(y(t) - y(0)) \rangle}{\langle (y(t) - y(0))^2 \rangle} = 2^{2H-1} - 1, \quad (3)$$

where the correlations are normalized. This relationship assumes obviously that the distribution of daily fluctuations $(y(t+1) - y(t))$ is symmetric with respect to its zero mean value.

From the point of view of economic signals, a value of H around 0.55 has been obtained for the USD/DEM ratio [19]. A review of exponent values for other currency ratios can be found in [20]. If different values of H are found in different regions of the signal, there is a good chance that the signal is multi-affine rather than self-affine [21]. These multi-affine signals can be described only in terms of an infinite set of exponents and their density distribution. Multi-affine signals are frequently described as *turbulent*, because the local velocities in turbulent systems exhibit a similar multiscaling behavior [22].

Through a *Detrended Fluctuation Analysis* (DFA) [19], we have put into evidence that the nature of the correlations (*i.e.* the H value) in the USD/DEM currency exchange rate can change with time. As a consequence, the USD/DEM evolution can be decomposed into successive persistent and antipersistent sequences [19] and multi-affine behaviors can be expected. More recently, it has been reported that the foreign exchange market effectively produces multi-affine signals [23–25].

In the present work, we perform a careful and more complete multi-affine analysis of the USD/DEM and JPY/USD exchange rates, and we give for the first time the so-called $h(\gamma)$-curve (see below) for such economic signals.

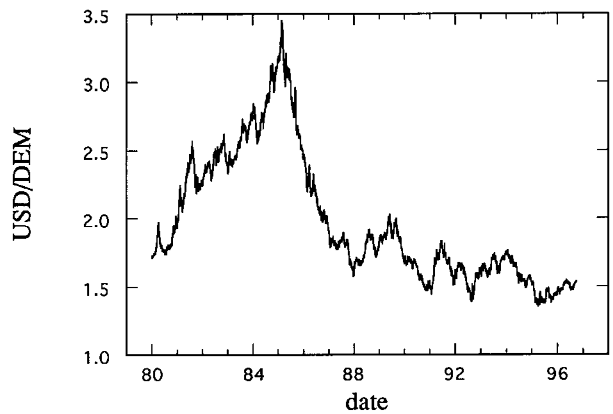

Fig. 1. The evolution of the USD/DEM exchange rate from Jan. 1, 1980 to Dec. 31, 1996.

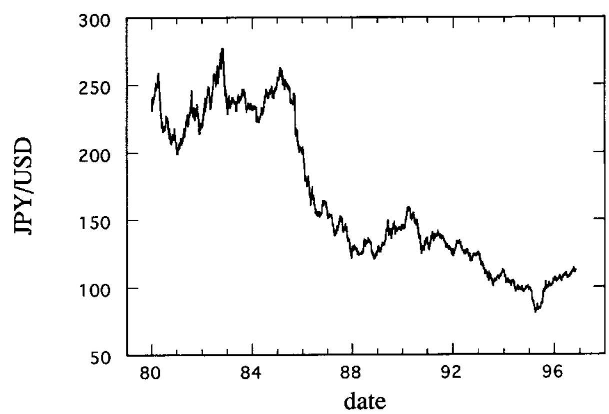

Fig. 2. The evolution of the JPY/USD exchange rate from Jan. 1, 1980 to Dec. 31, 1996.

2 Multi-affinity

A method for determining the multi-affinity of a time-dependent signal $y(t)$ is the so-called "q-th order height-height correlation function" [21] defined by

$$c_q(\tau) = \langle |y(t) - y(t')|^q \rangle_\tau, \quad \text{with} \quad \tau = |t - t'| \quad (4)$$

where only non-zero terms are taken into account in the average $\langle . \rangle$ over all couples of points $(y(t), y(t'))$. In fact, this measure takes the various moments q of the excursion of the signal. Assuming a power law scaling for the correlation function, the exponent $H(q)$ is defined through the relation

$$c_q(\tau) \sim \tau^{qH(q)}. \quad (5)$$

The signal $y(t)$ is multi-affine if $H(q)$ is a decreasing function of q [21]. For a self-affine signal, the $H(q)$ exponent is independent of q and is equal to the Hurst exponent of the signal. For a Brownian motion, the spectrum is expected to be $H(q) = 1/2$ for any $q > -1$ and to diverge for $q < -1$ [21]. One should note that the spectrum $H(q)$ is not defined for $q = 0$. If the signal $y(t)$ has too small height differences, one cannot calculate numerically the negative powers of $c_q(\tau)$ since the average of a negative power of a very small number leads to a divergence. But in most cases, the positive moments $(q > 0)$ strongly indicate whether the time-series is multi-affine or not [21,26].

3 Data

In the present case, $y(t)$ will be taken as the closing values at successive open banking days recorded in Brussels [27]. Figures 1 and 2 present respectively the evolution of the USD/DEM and JPY/USD exchange rates from Jan. 1, 1980 to Dec. 31, 1996. Week-end and holidays being non-banking days, only ≈ 261 data points are considered each year. Thus, about 4400 points are considered for each currency in the period investigated herein, covering 16 years. Let us point out that a possible week-end effect should not affect our results since small periodic events are averaged in equation (4) and do not change the measured value of $H(q)$.

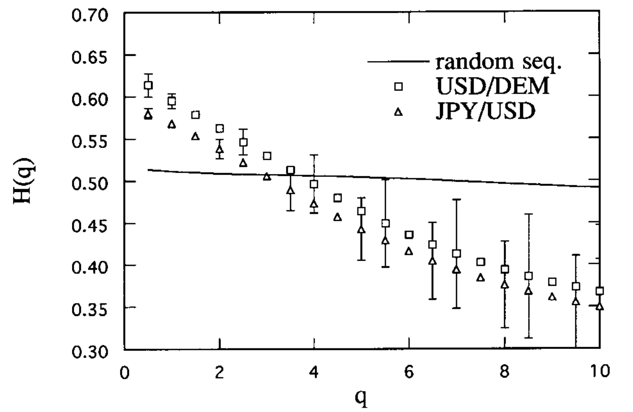

Fig. 3. The $H(q)$ spectrum for the currency exchange rates of Figures 1 and 2. Error bars are indicated. The continuous curve corresponds to the theoretical spectrum for a random walk.

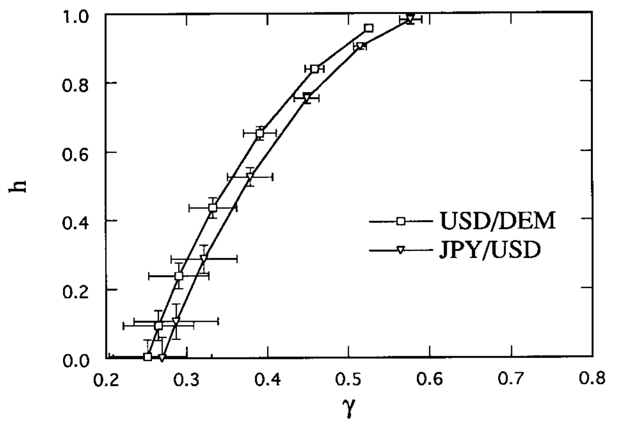

Fig. 4. The $h(\gamma)$-curve for both currency exchange rates of Figures 1 and 2. Error bars are indicated.

4 The H(q) spectrum

Figure 3 presents the $H(q)$ spectrum for the two currency exchange rates corresponding to Figures 1 and 2. Error bars are indicated for a few points in order to bring reliance to the discussion. One should note that the results for $q < 0$ lead to large uncertainties so that the numerical analysis is somewhat less relevant in this part of the spectrum at this time. A recent progress [28] has been made in order to obtain the negative moments of physiological signals. The latter technique is a modified wavelet analysis which will provide the measure of the negative moments in future work. In order to emphasize the typical uncertainties of the present technique, Figure 3 presents also the $H(q)$ spectra for a random walk ($H_q = 1/2$) of 4000 steps (continuous line). The random walk as well as self-affine signals made of 4400 points present all this small negative slope in the measured $h(q)$ spectrum. This is due to a finite-size effect.

The first observation to be made is that the spectrum $H(q)$ varies strongly with q for such exchange rates involving the USD. These huge variations of $H(q)$ are much larger than the finite-size effect such that they are not artefacts. Thus, the time-evolution of these currency exchange rates is clearly not self-affine but *multi-affine*. Therefore, there is a hierarchy of exponents characterizing the correlations in the economic system as for developed turbulence [22] in the physics of fluids. This was also suggested in [25].

5 The hierarchy of exponents h(γ)

The hierarchy of exponents can be modelled as follows. In order to rescale a multi-affine signal $y(t)$, one should use different scaling factors $H(q)$. Let us define the *local* scaling exponent γ in order to characterize the local singularity of the $y(t)$-signal

$$|y(t) - y(t+\tau)| \sim \tau^{\gamma}, \tag{6}$$

so that γ acts as a *local* roughness exponent of the $y(t)$ signal. One can define the number of points $N(\gamma)d\gamma$ that have an exponent in the range $(\gamma, \gamma + d\gamma)$. Such a density is assumed to scale with the time span τ used to probe the signal,

$$N_\gamma(\tau) \sim \tau^{-h(\gamma)} \tag{7}$$

like in multifractal objects [29]. The function $h(\gamma)$ is in fact the fractal dimension of the subset (like a Cantor dust [4,18]) of points having the same roughness local exponent γ. If the signal is self-affine, the local exponent γ is the same on every point such that $h(\gamma) = 1$. From [30], the following relations are found

$$\gamma(q) = \frac{d(qH_q)}{dq} \tag{8}$$

and

$$h(\gamma_q) = 1 + q\gamma(q) - qH_q. \tag{9}$$

The $h(\gamma)$ function is naturally adapted to describe multiaffine signals as the function $f(\alpha)$ describes multifractal objects [29]. However, the $h(\gamma)$-curve is not identical to the $f(\alpha)$-curve. The $f(\alpha)$-curve can be calculated from the $h(\gamma)$ function following some transformations fully described in [30]. However, the $h(\gamma)$ function is naturally better suited to describe multiaffine signals than $f(\alpha)$ [21,26,30].

The $h(\gamma)$-curve is drawn in Figure 4 for the two currency exchange rates analysed herein. Error bars are indicated. They are estimated following standard statistical data analysis [31]. The curves reach a maximum fractal dimension $h = 1$ for a γ_0 value which depends on the analyzed currency. The value γ_0 represents the Hurst exponent H value that one can calculate by considering $y(t)$ as a simple self-affine signal and neglecting the hierarchy of exponents. Thus, γ_0 represents a kind of a "zero-order" roughness exponent. Table 1 gives the values of γ_0 for the exchange rates analyzed. The value obtained for the USD/DEM ratio is consistent with thecoarse grained value

Table 1. Values of γ_0 and γ_{min} characterizing the hierarchy of exponents for both currency exchange rates studied in the present work.

exchanging rate	γ_0	γ_{min}
USD/DEM	0.57 ± 0.02	0.25 ± 0.06
JPY/USD	0.58 ± 0.02	0.27 ± 0.06

of H that we have obtained with the DFA technique [19]. The values of γ_0 are quite close to values obtained in [24] for the USD/FRF and JPY/FRF ratios.

The $h(\gamma)$-curves of Figure 4 seem also to vanish at some γ_{min} value. This value corresponds to the minimum Hurst exponent contained in the $y(t)$ signal. This exponent can be found by extrapolating $\gamma(q)$ for $q \to +\infty$. The γ_{min} values are listed in Table 1. The minimum Hurst exponent found herein seems to be roughly 0.25.

Since $\gamma_{min} \approx 0.25$, there is no white noise ($\gamma = 0$) component for the currency exchange rates studied herein. This value $\gamma_{min} \approx 0.25$ is surprisingly quite close to the Hurst exponent values obtained with the DFA method for European currency ratios like the DEM/BEF or NLG/BEF ratios [20]. It should be also noted that there exists a random (Brownian) component $\gamma = 1/2$ in each case. One can observe in Figure 4 that the set of points for the random component has the same fractal dimension $h \approx 0.91$ for both currency ratios.

One should also note that deterministic multiaffine functions [26] are characterized by a cap-convex $h(\gamma)$-curve (like $-\gamma^2$) such that a γ_{max} value can be also defined. In the present case, the part of the $h(\gamma)$-curve containing an hypothetic $\gamma_{max} > \gamma_0$ seems to be missing. This is due to the fact that c_q "fails to scale" for $q < 0$. Thus, one cannot extract the whole $h(\gamma)$-curve. This finding is quite similar to recent work on $f(\alpha)$-curve of the *Diffusion-Limited Aggregation* model (DLA) [32] or the so-called *self-similar left-sided multifractals* [33].

6 Discussion

Hereabove, we have put into evidence the multi-affine character of the evolution of currency exchange rates. This strongly supports the idea that economic system description needs laws more complicated than simple powers [34]. This also suggests that a physical modelling of currency evolution is possible within *e.g.* the framework of the physics of fluid turbulence. The reader also interested by the implications of our findings from the point of view of the Lévy distributions can be referred to [24].

Moreover, we have obtained the $h(\gamma)$-curves which describe the hierarchy of exponents for "turbulent economic excursions". To our knowledge, it is the first time that this curve is obtained for foreign exchange markets. By analogy with the $f(\alpha)$ function for multifractals, h can be *formally* associated to some quantity of information, γ associated to a free energy and q to the inverse of a temperature [35].

With respect to the present results, the information is minimum for an antipersistent behavior $\gamma = \gamma_{min}$ and is maximum for a persistent behavior $\gamma = \gamma_0$.

Notice that in the *Efficient Market Theory* [36] beloved by some economists, the argument goes that new informations occur stochastically, whence one should observe (according to this theory) economic indices like currency exchange rates follow random walk laws. In contrast, the hierarchy of exponents herein put into evidence seems to demonstrate that there is a multiplicity of information levels, which could be taken into account in order to describe whatever economical signal evolution.

7 Summary

The multi-affine method is useful for analyzing the nature of high order long range correlations in economic systems. The present work has demonstrated the multi-affine character of the daily closing values of USD/DEM and JPY/USD currency exchange rates on the Brussels market. The $h(\gamma)$-curve describing the density of the data local roughness has been extracted. This result suggests that a physical modelling of exchange rate time-evolutions is possible through *e.g.* models of turbulence. The non-trivial $h(\gamma)$ curve demonstrates by analogy with multifractal ideas that there is a multiplicity of information levels such that the efficient market theory is oversimplified.

The powerfulness of the multi-affine analysis in detecting *non-efficiency* can be implemented in various market investigations for hedging positions of portfolios. Further works should concern other currency ratios and other markets.

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